Callable Inverse Swap Model

A Callable Inverse Floating Rate Swap is a forward swap agreement with an option of canceling the swap each year starting from several years in future. The deal is priced with a two factor Black-Karasinski model.

The Black-Karasinski class of models assumes the short term interest rates to be log-normally distributed. The spreadsheet mode used for the deal pricing has a hard-coded term structure of the mean reversion and volatility parameters, constructed as Chebyshev polynomials.

The calibration procedure takes only an interest rate curve as input (ignoring volatility surfaces) and results in adjusting the "alpha" parameter of the model. To test the calculations over a range of parameters, we used the "piece-wise constant parametrization" mode.

The Black-Karasinski model is set with zero mean reversion. The zero mean reversion makes it easy to perform simulations on a recombining binomial tree.

Assume that short term interest rate process, $\{r_t \mid t \geq 0\}$, satisfies, under the risk neutral probability measure, a SDE of Black-Karasinski form,

$$d \log r_t = (\theta(t) - a \log r_t)dt + \sigma_r dW_t^r, \qquad t \geq 0,$$

where

- $\{W_t^r \mid t \geq 0\}$ denotes standard Brownian motion,
- σ_r is the volatility,
- a, with $a > 0$, is the mean reversion,
- $\theta(t)$ is chosen to match the initial term structure of zero coupon bond prices.

Our approach towards building a tree for the short-term interest rate process, $\{r_t \mid t \in [0,T]\}$, is based on the single-factor tree construction technique described in [Hull, 1994a]. Specifically let

$$\log r_t = (\log r_t - \tilde{r}_t) + \tilde{r}_t ,$$

where the process $\{\tilde{r}_t \mid t \in [0,T]\}$ satisfies the SDE

$$\begin{cases} d\tilde{r}_t = -a\tilde{r}_t + \sigma_r dW_t^r , \\ \tilde{r}_0 = 0. \end{cases}$$

Then

$$\begin{cases} d\alpha_t = (\theta_t - a\alpha_t)dt, \\ \alpha_0 = \log r_0 , \end{cases}$$

where $\alpha_t = \log r_t - \tilde{r}_t$. We first build a tree for the process $\{\tilde{r}_t \mid t \in [0,T]\}$ as described below.

To provide the ability to control interest rates for our testing, the interest rate curve was constructed as a flat yield curve at a prescribed yield level. One set of the tests used the interest rate term structure as generated.

A few tests of internal consistency of the model raise some concerns. The results of symmetry tests cannot be accounted for by numerical truncation errors.

Fortunately, these discrepancies are not important for the deal under consideration. The reason is that the price dynamics that affect the deal is driven by essentially one factor process. The history of tests of one factor driven processes supports the validity of pricing.

However, extra caution is needed when the model is used for pricing deals that are driven by essentially two factor random processes. Eventually, the indicated inconsistencies in the model need to be eliminated.

Find more callable derivatives at:

https://finpricing.com/curveVolList.html